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User Documentation for a Liver Constitutive Model

by John D. Clayton

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14. ABSTRACT
This technical note provides user documentation for a constitutive model of the liver implemented as a user-material subroutine in the LS-DYNA software. History variables, material parameters, and user-specified controls are identified. Syntax for the material model block in the LS-DYNA input file is explained. A sample input file for deformation of a single hexahedral element is provided with recommended values of parameters.

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[1]Clayton J. Modeling dynamic impact, shock waves, and injury in liver tissue with a constrained mixture theory. Biomechanics and Modeling in Mechanobiology. 2025; doi.org/10.1007/s10237-025-01990-3.

1. Introduction

This technical note provides user documentation for a constitutive model of the liver[1] implemented for 3-D finite element (FE) simulations as a user-material subroutine in the LS-DYNA software.[2] Potential users should also consult the LS-DYNA software manuals, currently available at https://lsdyna.ansys.com/manuals/, for instructions on how to compile and execute generic user-material subroutines in `LS-DYNA`. User documentation for a constitutive model of lung tissue mechanics and injury,[3,4] similarly implemented in LS-DYNA, was provided in previous reports.[5,6]

The liver constitutive model is implemented in the `FORTRAN` programming language as a user-defined material in the `LS-DYNA` software, identified by user-defined material number 49. The global subroutine for the constitutive update is named `umat49`. This file, in turn, is inserted in the master file `dyn21umats.f` and recompiled with the source prior to execution, as explained in the `LS-DYNA` documentation.[2] The present implementation is limited to explicit dynamics calculations. As such, no tangent stiffness tensor that would be needed for implicit calculations is required. Per standard conventions of the software, in each time increment $t \longrightarrow t + \Delta t$, the local deformation gradient $\boldsymbol{F}$ is supplied at the end of the time step, as are values of deformation rate $\boldsymbol{d}$, material properties, and state variables at the beginning of the step. The user-material subroutine calculates, at each integration point, the Cauchy stress tensor $\boldsymbol{\sigma}$ and the state variables at the end of the time step, including temperature θ. To date, only isothermal and adiabatic simulations (i.e., no heat conduction) have been tested. The model has been verified only for `*ELEMENT_SOLID` continuum FEs; shell elements are not supported.

The objective of this note is to aid transition of a liver constitutive model to applied research on torso injury mechanics[7] with the `LS-DYNA` software. The reader is referred to the original journal article[1] and its supplemental material for details on underlying theory and computational methods that are not reproduced in this note. Prior references on soft-tissue mechanics and injury by the author's research group culminating in the present constitutive model are widely available.[8–14] Software in usable `*.f` format (e.g., `dyn21umats.f`) and input files in usable `*.k` format can be provided to collaborators holding valid `LS-DYNA` licenses upon request. A stand-alone `FORTRAN` program similar to that described by Clayton and Lloyd[15] has also been implemented for verification testing of the liver model constitutive routines. This program, not documented in the present note but available upon request, can be used in lieu of `LS-DYNA` during development and testing stages of generic user-material model subroutines.

Syntax for the material model block in the LS-DYNA input file is explained in Section 2. History variables, material parameters with recommended values, and user-specified controls for the material input cards are identified. A sample input file for uniaxial (i.e., 1-D) deformation of a single 3-D hexahedral element is provided in Section 3. A list of notation defining acronyms and symbols supplementing the theoretical reference[1] then follows the bibliography of Section 4.

2. Variables and Syntax

2.1 History Variables

The `LS-DYNA` implementation stores a total of 64 history variables in the `hsv` vector. These are identified as follows:

HISTORY VARIABLES

`hsv` (1)	temperature θ
`hsv` (2)	matrix damage order parameter $\bar{D}$
`hsv` (3)	fiber damage order parameter D_k $(k = 1)$
`hsv` (4)	matrix injury measure $\bar{I}$
`hsv` (5)	fiber injury measure I_k $(k = 1)$
`hsv` (6)–(30)	internal variables and energy for matrix viscoelasticity
`hsv` (31)–(55)	internal variables and energy for fiber viscoelasticity
`hsv` (56)–(64)	components of deformation gradient $\boldsymbol{F}$

The first five history variables, calculated in the user subroutine `umat49`, and the last nine (i.e., $\boldsymbol{F}$; see Clayton[16] or any continuum mechanics reference) calculated by the `LS-DYNA` main routine, are usually of keen interest for postprocessing. The other 50 possible variables are required by `umat49` to integrate the kinetic equations for viscoelasticity for all potential modes and relaxation times; these variables (e.g., $\{\boldsymbol{Q}, \boldsymbol{\Gamma}, \Psi\}$) are likely of low interest to the user in the context of postprocessing and results visualization.

2.2 Input File Syntax

User-specified input for the material model comprises 48 local material constants read from the input file and stored in the `cm` vector. These are supplied in the input file block as six additional lines or "cards", with eight constants per card, that follow the first two obligatory cards for any material model.

See the end of this technical note and Table 1 of the main journal article[1] for notation and more thorough material property definitions. Suggested values in what follows correspond to perfused liver.[1] Though any consistent set of units can be used, recommended SI units (not all independent) are $\{\mathrm{kg}, \mathrm{m}, \mathrm{sec}, \mathrm{N}, \mathrm{Pa}, \mathrm{K}\}$. Input in `LS-DYNA` syntax (e.g., in the `*.k` file) is as follows:

`*MAT_USER_DEFINED_MATERIAL_MODELS`

CARD 1

`MID`	material identification: unique 8-digit number
`RO`	initial mass density of the mixture $\rho_0 = \sum_\alpha n_0^\alpha \rho_{R0}^\alpha \approx \sum_\alpha n_0^\alpha \varrho_{R0}^\alpha$
`MT`	material number for user-defined subroutine call (49, fixed)
`LMC`	length of material constant array (48, fixed)
`NHV`	number of user history variables excluding $\boldsymbol{F}$ (55, fixed)
`IORTHO`	material is not orthotropic (0, fixed)
`IBULK`	card 3 location for bulk modulus in time step calculation (1, fixed)
`IG`	card 3 location for shear modulus in time step calculation (2, fixed)

CARD 2

`IVECT`	vectorized material model not implemented (0, fixed)
`IFAIL`	independent failure flag not implemented (0, fixed)
`ITHERM`	temperature implemented (1, fixed)
`IHYPER`	deformation gradient $\boldsymbol{F}$ stored in `hsv` vector (1, fixed)
`IEOS`	separate equation of state not implemented (0, fixed)

CARD 3

BULK	mixture isentropic bulk modulus used by LS-DYNA in Δt estimate
SHEAR	mixture isotropic shear modulus used by LS-DYNA in Δt estimate
muS	matrix shear modulus μ_{S}
muk	fiber shear modulus μ_k
kk	fiber exponential stiffening factor k_k
theta0	reference and initial temperature θ_0
dJ0s	initial volume change for solid $J^{\mathrm{s}}_{0*} - 1$
dJ0f	initial volume change for fluid $J^{\mathrm{f}}_{0*} - 1$

CARD 4

n0	solid volume fraction n^{s}_0
rhoR	solid isolated mass density $\varrho^{\mathrm{s}}_{\mathrm{R0}}$
B0	solid isothermal bulk modulus B^{s}_{θ}
Bp	solid pressure derivative of bulk modulus $B^{\mathrm{s}}_{\theta\mathrm{p}}$
kv	solid exponential bulk stiffening factor $k^{\mathrm{s}}_{\mathrm{V}}$
cv	solid specific heat at constant strain/volume $c^{\mathrm{s}}_{\epsilon}$
gruneis	solid Grüneisen parameter γ^{s}_0
pR0	ambient reference pressure p^{α}_{R} for all phases (e.g., 1 atm)

CARD 5

n0	fluid volume fraction n^{f}_0
rhoR	fluid isolated mass density $\varrho^{\mathrm{f}}_{\mathrm{R0}}$
B0	fluid isothermal bulk modulus B^{f}_{θ}
Bp	fluid pressure derivative of bulk modulus $B^{\mathrm{f}}_{\theta\mathrm{p}}$
kv	fluid exponential bulk stiffening factor $k^{\mathrm{f}}_{\mathrm{V}}$
cv	fluid specific heat at constant strain/volume $c^{\mathrm{f}}_{\epsilon}$
gruneis	fluid Grüneisen parameter γ^{f}_0
pcav	fluid cavitation pressure: if nonzero, max allowed value of $-p^{\mathrm{f}}_{\mathrm{V}}/n^{\mathrm{f}}_0$

CARD 6

tmpflag	set to 1 for (adiabatic) temperature update, or set to 0 for isothermal

`dmgflag`	set to 1 to enable damage modeling, or set to 0 for no tissue damage
`Ematrix`	matrix cohesive energy $\bar{E}_{\mathrm{C}}$
`Efiber`	fiber cohesive energy $E_{\mathrm{C}k}$
`viscdmg`	viscosity time scale for fracture kinetics $\hat{\nu}_{\mathrm{D}}$
`vartheta`	phase-field fracture degradation exponent $\bar{\vartheta} = \vartheta_k$
`erem`	remnant strain at rupture $\bar{\epsilon} = \tilde{\epsilon}_k$
`ralpha`	non-Euclidean metric scaling factor $\bar{r} = \tilde{r}_k$

CARD 7

`injflag`	set to 1 to enable injury modeling, or set to 0 for no injury prediction
`dJtilde`	reference volume change for injury model $\tilde{J}_0^{\mathrm{s}} - 1$
`npower`	user-specified solution control variable for damage and injury update
`numiter`	user-specified convergence iterations for damage and injury update*
`cl`	linear shock viscosity coefficient c_{l}
`cq`	quadratic shock viscosity coefficient c_{q}
`dDmax`	user-specified max allowed $\dot{\bar{D}}\Delta t$ and $\dot{D}_k\Delta t$ in single increment Δt
`tenflag`	set to 1 for dilatational energy contribution to matrix damage, else 0

CARD 8

`betaM1`	matrix viscoelastic stiffening factor $\beta_{\mathrm{S}1}$
`betaM2`	matrix viscoelastic stiffening factor $\beta_{\mathrm{S}2}$
`tauM1`	matrix viscoelastic relaxation time $\tau_{\mathrm{S}1}$
`tauM2`	matrix viscoelastic relaxation time $\tau_{\mathrm{S}2}$
`betaF1`	fiber viscoelastic stiffening factor $\beta_{\Phi k,1}$
`betaF2`	fiber viscoelastic stiffening factor $\beta_{\Phi k,2}$
`tauF1`	fiber viscoelastic relaxation time $\tau_{\Phi k,1}$
`tauF2`	fiber viscoelastic relaxation time $\tau_{\Phi k,2}$

*Reducing `numiter` can speed up simulations but may decrease the accuracy of injury predictions.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	MT	LMC	NHV	IORTHO	IBULK	IG
Required	user-prescribed	see text	**49**	**48**	**55**	**0**	**1**	**2**
Type	A8	F	I	I	I	I	I	I

Card 2	1	2	3	4	5	6	7	8
Variable	IVECT	IFAIL	ITHERM	IHYPER	IEOS			
Required	**0**	**0**	**1**	**1**	**0**			
Type	I	I	I	I	I			

Card 3	1	2	3	4	5	6	7	8
Variable [units]	BULK [Pa]	SHEAR [Pa]	muS [Pa]	muk [Pa]	kk [none]	theta0 [K]	dJ0s [none]	dJ0f [none]
Suggested	2.67E+09	1.0E+05	1.0E+03	1.0E+05	1.0E-06	310.0	5.038E-06	-5.126E-06
Type	F	F	F	F	F	F	F	F

Card 4	9	10	11	12	13	14	15	16
Variable [units]	n0 [none]	rhoR [kg/m^3]	B0 [Pa]	Bp [none]	kv [none]	cv [J/kg K]	gruneis [none]	pR0 [Pa]
Suggested	0.50	1060.0	2.66E+09	8.0	6.0	3.51E+03	0.114	1.013E+05
Type	F	F	F	F	F	F	F	F

Card 5	17	18	19	20	21	22	23	24
Variable [units]	n0 [none]	rhoR [kg/m^3]	B0 [Pa]	Bp [none]	kv [none]	cv [J/kg K]	gruneis [none]	pcav [Pa]
Suggested	0.50	1060.0	2.61E+09	12.0	0.0	3.58E+03	0.160	9.7E+06
Type	F	F	F	F	F	F	F	F

Card 6	25	26	27	28	29	30	31	32
Variable [units]	tmpflag [none]	dmgflag [none]	Ematrix [Pa]	Efiber [Pa]	viscdmg [sec]	vartheta [none]	erem [none]	ralpha [none]
Suggested	1.0	1.0	2.27E+04	2.27E+04	0.05	2.0	0.2	2.0
Type	F	F	F	F	F	F	F	F

Card 7	33	34	35	36	37	38	39	40
Variable [units]	injflag [none]	dJtilde [none]	npower [none]	numiter [none]	cl [none]	cq [none]	dDmax [none]	tenflag [none]
Suggested	1.0	5.038E-06	3.0	100.0	0.06	1.5	1.0	1.0
Type	F	F	F	F	F	F	F	F

Card 8	41	42	43	44	45	46	47	48
Variable [units]	betaM1 [none]	betaM2 [none]	tauM1 [sec]	tauM2 [sec]	betaF1 [none]	betaF2 [none]	tauF1 [sec]	tauF2 [sec]
Suggested	20.0	150.0	5.0E-02	1.0E-03	1.0	0.0	1.0E-03	1.0E-03
Type	F	F	F	F	F	F	F	F

3. Sample Input File

An example input file invoking the user-material model in `LS-DYNA` syntax follows subsequently. The first unique feature of this file is the material history outputs requested for the first five entries of `hsv`—θ, $\bar{D}$, D_k, $\bar{I}$, I_k—as well as all nine components of the deformation gradient $\boldsymbol{F}$. The second unique feature is specification of the user-defined (liver) material model and parameters as described in Section 2 of this technical note. Other standard features of the `LS-DYNA` input file are documented in the manuals.[2]

The input file for a one-element mesh is provided in what follows next. Boundary conditions compress a single hexahedral element by 10% in the X_1-direction over a total time interval of 1 s. The space–time domain, loading conditions, and material properties can be adjusted, if need be, to model various 1-D problems as analyzed in Section 4 of the main journal article.[1] An exception is that damage and injury to the fluid are not addressed by the `LS-DYNA` source code and input file supplied here. These features were included in a previous iteration of computer code. They are omitted now for simplicity and computational efficiency since immense pressures, outside a pragmatic range, were predicted necessary to damage and injure the fluid phase (e.g., blood cells) relative to the much less resilient solid tissue matrix and fibers.[1] Note also that a different approach is needed to solve the Rankine–Hugoniot equations analytically[11,17] versus numerically in `LS-DYNA`. In the latter, shock width is finite due to standard shock viscosity. See Section 4.1 of the main journal article[1] for details.

```
*KEYWORD
*TITLE
single 8-node hexahedral finite element of liver: user-defined material 49
*CONTROL_TERMINATION
       1.0
*DATABASE_BINARY_D3PLOT
    1.0E-1
$--------+---------+---------+---------+---------+---------+---------+---------+
*DATABASE_EXTENT_BINARY
$    NEIHP     NEIPS    MAXINT    STRFLG    SIGFLG    EPSFLG    RLTFLG    ENGFLG
        64        64         3         1         1         1         1         1
$   CMPFLG    IEVERP    BEAMIP     DCOMP      SHGE     STSSZ    N3THDT   IALEMAT
         0         0         0         1         1         1         2         1
$  NINTSLD   PKP_SEN      SCLP     HYDRO     MSSCL     THERM    INTOUT    NODOUT
         1         0       1.0         0         0         0
$--------+---------+---------+---------+---------+---------+---------+---------+
*DEFINE_MATERIAL_HISTORIES_NAMES
History,1,49,1,1
THETA
History,2,49,1,1
DMAT
History,3,49,1,1
DFIB
History,4,49,1,1
IMAT
History,5,49,1,1
IFIB
History,56,49,1,1
F11
History,57,49,1,1
F21
History,58,49,1,1
F31
History,59,49,1,1
F12
History,60,49,1,1
F22
History,61,49,1,1
F32
History,62,49,1,1
F13
History,63,49,1,1
F23
History,64,49,1,1
F33
*PART
single 3D element
$--------+---------+---------+---------+---------+---------+---------+---------+
$      PID     SECID       MID     EOSID      HGID      GRAV    ADPOPT      TMID
         1         1         1
*SECTION_SOLID
$    SECID    ELFORM       AET
         1
$--------+---------+---------+---------+---------+---------+---------+---------+
*MAT_USER_DEFINED_MATERIAL_MODELS
$--------+---------+---------+---------+---------+---------+---------+---------+
$      MID        RO        MT       LMC       NHV    IORTHO     IBULK        IG
         1    1060.0        49        48        55         0         1         2
$    IVECT     IFAIL    ITHERM    IHYPER      IEOS
         0         0         1         1         0
$     BULK     SHEAR       muS       muk        kk    theta0      dJ0s      dJ0f
   2.67E+9  1.00E+5   1.00E+3   1.00E+5    1.0E-6     310.0  5.038E-6 -5.126E-6
$solid: n0      rhoR        B0        Bp        kv        cv   gruneis       pR0
      0.50    1060.0   2.66E+9       8.0       6.0   3.51E+3     0.114  1.013E+5
$fluid: n0      rhoR        B0        Bp        kv        cv   gruneis      pcav
      0.50    1060.0   2.61E+9      12.0       0.0   3.58E+3     0.160    9.7E+6
$  tmpflag   dmgflag   Ematrix    Efiber   viscdmg  vartheta      erem    ralpha
       1.0       1.0   2.27E+4   2.27E+4      0.05       2.0       0.2       2.0
$  injflag   dJtilde    npower   numiter        cl        cq     dDmax   tenflag
       1.0  5.038E-6       3.0      100.      0.06       1.5       1.0       1.0
$   betaM1    betaM2     tauM1     tauM2    betaF1    betaF2     tauF1     tauF2
      20.0     150.0    5.0E-2    1.0E-3       1.0       0.0    1.0E-3    1.0E-3
```

```
$--------+---------+---------+---------+---------+---------+---------+---------+
*NODE
$------+---------------+---------------+---------------+-------+-------+-------+
$    NID               X               Y               Z      TC      RC
       1              0.              0.              0.       7       7
       2              1.              0.              0.       5       0
       3              1.              1.              0.       3       0
       4              0.              1.              0.       6       0
       5              0.              0.              1.       4       0
       6              1.              0.              1.       2       0
       7              1.              1.              1.       0       0
       8              0.              1.              1.       1       0
*SET_NODE_GENERAL
$--------+---------+---------+---------+---------+---------+---------+---------+
$#     SID       DA1       DA2       DA3       DA4    SOLVER
         1         0         0         0         0      MECH
$#  OPTION        E1        E2        E3        E4        E5        E6        E7
      NODE         1         4         5         8
*SET_NODE_GENERAL
$--------+---------+---------+---------+---------+---------+---------+---------+
$#     SID       DA1       DA2       DA3       DA4    SOLVER
         2         0         0         0         0      MECH
$#  OPTION        E1        E2        E3        E4        E5        E6        E7
      NODE         2         3         6         7
*SET_NODE_GENERAL
$--------+---------+---------+---------+---------+---------+---------+---------+
$#     SID       DA1       DA2       DA3       DA4    SOLVER
         3         0         0         0         0      MECH
$#  OPTION        E1        E2        E3        E4        E5        E6        E7
       ALL
$--------+---------+---------+---------+---------+---------+---------+---------+
*SET_PART
$--------+---------+---------+---------+---------+---------+---------+---------+
$#     SID       DA1       DA2       DA3       DA4    SOLVER
         1         0         0         0         0      MECH
$#     PID
         1
*ELEMENT_SOLID
$------+-------+-------+-------+-------+-------+-------+-------+-------+-------+
$    EID     PID      N1      N2      N3      N4      N5      N6      N7      N8
       1       1       1       2       3       4       5       6       7       8
$--------+---------+---------+---------+---------+---------+---------+---------+
*BOUNDARY_PRESCRIBED_MOTION_SET
$#  TYPEID       DOF       VAD      LCID        SF       VID     DEATH     BIRTH
         1         1         0         1       1.0
*BOUNDARY_PRESCRIBED_MOTION_SET
$#  TYPEID       DOF       VAD      LCID        SF       VID     DEATH     BIRTH
         2         1         0         2     -10.0
*BOUNDARY_PRESCRIBED_MOTION_SET
$#  TYPEID       DOF       VAD      LCID        SF       VID     DEATH     BIRTH
         3         2         0         1       1.0
*BOUNDARY_PRESCRIBED_MOTION_SET
$#  TYPEID       DOF       VAD      LCID        SF       VID     DEATH     BIRTH
         3         3         0         1       1.0
*DEFINE_CURVE
$--------+---------+---------+---------+---------+---------+---------+---------+
$     LCID      SIDR       SFA       SFO      OFFA      OFFO    DATTYP
         1
$-------------------+-------------------+
$                 A1                  O1
                  0.             0.0e-04
                  1.             0.0e-04
*DEFINE_CURVE
$--------+---------+---------+---------+---------+---------+---------+---------+
$     LCID      SIDR       SFA       SFO      OFFA      OFFO    DATTYP
         2
$-------------------+-------------------+
$                 A1                  O1
                  0.             1.0e-02
                  1.             1.0e-02
*END
```

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List of Symbols, Abbreviations, and Acronyms

TERMS

1-D	one-dimensional
3-D	three-dimensional
FE	finite element
SI	International System of Units

MATHEMATICAL SYMBOLS

$(\square)^\alpha$	quantity for constituent or phase α
$(\square)^\mathrm{s}$	quantity for solid constituent
$(\square)^\mathrm{f}$	quantity for fluid constituent
$(\square)_k$	quantity for fiber family k
$(\square)_\mathrm{D}$	quantity for damage model
$(\square)_\mathrm{F}$	quantity for fiber hyperelasticity
$(\square)_\mathrm{R}$	quantity for isolated (i.e., real) constituent
$(\square)_\mathrm{S}$	quantity for deviatoric response
$(\square)_\mathrm{V}$	quantity for volumetric response
$(\square)_\Gamma$	quantity for matrix viscoelasticity
$(\square)_\Phi$	quantity for fiber viscoelasticity
$(\square)_0$	quantity typically defined in global reference configuration
$(\square)_*$	quantity in local reference state where properties measured
$(\square)_\star$	quantity measuring transition from state at t_* to current t
$(\dot{\square})$	material time derivative with respect to mixture
B_θ	isothermal bulk modulus
$B_{\theta\mathrm{p}}$	pressure derivative of isothermal bulk modulus
c_ϵ	specific heat per unit mass at constant strain
c_l	linear bulk viscosity coefficient
c_q	quadratic bulk viscosity coefficient
$\boldsymbol{C}$	right Cauchy–Green tensor
$\boldsymbol{d}$	spatial deformation rate tensor
D	order parameter or damage measure for fiber fracture
$\bar{D}$	order parameter for matrix fracture or fluid cavitation

E_{C}	cohesive energy of fibers per unit reference volume
$\bar{E}_{\mathrm{C}}$	cohesive energy of matrix or fluid per unit reference volume
$\boldsymbol{F}$	deformation gradient
I	scalar injury measure for fiber family
$\bar{I}$	scalar injury measure for matrix or fluid
J	Jacobian determinant of deformation gradient
$\tilde{J}_0$	reference determinant for local state used in injury model
k	fiber exponential stiffening constant
k_{V}	volumetric exponential stiffening constant
n_0	referential constituent volume fraction
$n_\star$	mapped constituent volume fraction
p	Cauchy pressure
$\boldsymbol{Q}$	conjugate stress to viscoelastic configurational state variable
$\tilde{r}$	Finsler metric exponential constant for fibers
$\bar{r}$	Finsler metric exponential constant for matrix or fluid
t	time
$\boldsymbol{x}$	spatial (i.e., Eulerian) coordinates
$\boldsymbol{X}$	referential (i.e., Lagrangian or material) coordinates
β	viscoelastic stiffness factor
$\bar{\epsilon}$	remnant strain for matrix
$\tilde{\epsilon}$	remnant strain for fibers
γ_0	Grüneisen scalar
$\boldsymbol{\Gamma}$	viscoelastic configurational state variable
μ	shear modulus
ν	viscosity or kinetic parameter
θ	temperature
ϑ	damage degradation exponent
ρ	spatial mass density
ρ_0	referential mass density
ϱ_{R0}	real mass density of isolated constituent at 1 atm
$\boldsymbol{\sigma}$	total Cauchy stress tensor
τ	viscoelastic relaxation time
Ψ	Helmholtz free energy per unit volume

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