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An Analytical Approach to Level-Crossing-Based Sampling

by Vinod K. Mishra

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Vinod K. Mishra
DEVCOM Army Research Laboratory

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ORCID: Vinod Mishra, 0000-0001-9432-9082

14. ABSTRACT
Signals (e.g., communication, radar, and electrocardiogram) in general are analyzed using discrete time methods. Recently, advantages of continuous time methods for machine learning have been observed. This approach is nonuniform in time and uses sampling based on level-crossing. In this report we present an analytical model for the latter approach, which is also known as a level-crossing-based digital sampling process.

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Network, Cyber, and Computational Sciences; discrete-time signals; continuous-time signals; level-crossing; uniform time sampling; cubic curve

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Vinod K. Mishra	(520) 691-5761

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1. Introduction

There are many types of signals (e.g., communication, sensing, radar) with varied characteristics across a wide range in the tactical arena. For example, radar chirp signals have very complex time and frequency dependence. Applicable signal processing approaches consist of the following:

- Discrete-time digital signal processing (DT-DSP): Uses periodic discrete time sampling.
- Continuous-time digital signal processing (CT-DSP): Uses exact quantized amplitude and continuous time.

CT-DSP is relatively new and can potentially contribute to areas such as control systems, compressive sensing, and spiking neural networks (SNNs). In the current work, an analytical approach to calculate the level-crossing of these signals will be presented.

2. Continuous-Time Digital Signal Processing

DT-DSP is the standard and well-developed approach to signal processing. It is governed by well-known concepts and methods involving Nyquist rate, sinc (short for cardinal sine) functions, etc. The other approach to signal processing is CT-DSP,[1–8] which is an emerging subfield of signal processing. It has time domain properties like analog signal processing (asynchronous, no clock) with the benefits of DSP without the limitations of conventional sampling. CT-DSP has many advantages over conventional DT-DSP:

- The input continuous time signal is quantized when it equals a voltage level, resulting in no inherent quantization error. Subsequent power savings can be as high as 10 times in some cases.
- The sampling frequency of the input continuous time signal is proportional to its slope, e.g., a 2.5-s electrocardiogram (ECG) dataset requires only 225 samples compared to 1250 samples for conventional digital signal processing. Fewer samples result in less data processing and lower energy.
- CT-DSP does not use uniform time sampling, so it has no frequency aliasing.
- CT-DSP uses adaptive sampling, so an input signal is only captured when it crosses a threshold level. For signals whose important amplitudes are known as a priori, this can lead to significant power savings.
- The input signal is captured at the exact time and amplitude at which it crosses a threshold level. This accuracy allows the potential for

- High signal-to-noise and distortion (SINAD) reconstruction of level-crossing sampled (LCS) signals. LCS is a sequence of Dirac delta functions exactly representing the time and amplitude of all detected level-crossings.
- Higher SINAD values than conventional methods for level-crossing delta (LCD) signals as it counterbalances their harmonic distortion. LCD is a step function consisting of adjoining rectangular pulses.

The time accuracy of the sampling benefits control systems and allows accurate detection of the transient signals. This is accomplished by giving information at the time of a transient, instead of waiting for the next clock tick. Both CT-DSP and analog signal processing are asynchronous (no clock), but the former has a nonuniform time step. Quantization occurs when the input signal exactly equals a voltage level, resulting in no inherent quantization error. CT-DSP systems using the simplest reconstruction technique give signal-to-quantization noise ratios (SQNRs) significantly better than discrete time systems limited by quantization noise:

$$SQNR(n) = 6.02n + 1.76 \text{ dB}.$$

Machine learning (ML) models in general are expensive to train and highly dependent on the number of samples in the input. Using CT-DSP methods in neural network analysis of signals needs fewer data samples. Some of the reasons illustrating this point are given below:

- Discrete time data contains no knowledge of the contents (e.g., information entropy) of the dataset.
- Compressive sensing techniques can significantly reduce data sizes; however, they typically make assumptions about the data's characteristics.
- ML methods applied to the continuous time signals have the potential to significantly reduce the number of data points needed for sensing signals. Reducing the number of samples can exponentially reduce the cost of model training and deployment.

The CT-DSP methods have been used to analyze ECG signals[9–13] and SNNs,[14–18] and have improved understanding and applications over traditional methods.

3. Level-Crossing of a Continuous Time Signal

CT-DSP has discrete amplitude and continuous time, and it emphasizes accuracy timing over that in amplitude. It is characterized by equivalent times t_k, when the input signal exactly equals a discrete amplitude and there is no quantization error compared to DSP. It is nonlinear, so the signal reconstruction must be done before any processing. The continuous time sample point is a vector given as

- A time stamp and amplitude level (t_k, a_k) or
- A time difference and an amplitude level $(t_k - t_{k-1}, a_k)$. The time steps are fixed for the DT-DSP but are nonuniform for the CT-DSP. Input signal vectors in such a system can be characterized in either of two ways:
  - (t_k, a_k), where t_k= level-crossing timestamp, or
  - $(\Delta t_k,\ a_k)$, where $\Delta t_k = t_{k+1} - t_k$= time difference between level-crossing timestamps, and $a_k = n_k \Delta \mathrm{V}$ = amplitude level (n_k = an integer, and $\Delta \mathrm{V}$ = the distance between voltage levels).
- For continuous time, $a_k(t_k)$ = the amplitude level for level-crossing at time t_k, $x_{CT}(t_k) = a_k(t_k)$
- For discrete time, $t(k) = t_k, \quad x_{DT}(k) = x(kT)$

The following relation connects a discrete time signal to a corresponding continuous time signal.

$$x_d(t) = \sum_n x_d[n] p(\tau_0 + n\Delta t)$$

There are several analytical functions used for *p*-function depending on the nature of the signals. For band-limited signals, the raised cosine function is used widely. Regular level-crossing values in a continuous time signal correspond to nonuniform time values in $x_d(t)$.

4. A Mathematical Model of Level-Crossing-Based Sampling

4.1 Signal Represented as Piecewise Quadratic Functions

Let s_i denote the levels given as a quadratic at time t_i. Then the levels can be expressed as

$$s_1 = at_1^2 + bt_1 + c$$

$$s_2 = at_2^2 + bt_2 + c$$

$$\ldots\ldots$$

$$s_n = at_n^2 + bt_n + c$$

The first equation can be rewritten as

$$t_1^2 + \frac{b}{a} t_1 - \left(\frac{s_1 - c}{a}\right) = 0$$

We get

$$\left(t_1 + \frac{b}{2a}\right)^2 - \left\{\frac{s_1 - c}{a} + \left(\frac{b}{2a}\right)^2\right\} = 0$$

The solution for the time corresponding to the crossing of the first level becomes

$$t_1 = -\frac{b}{2a} + \sqrt{\frac{s_1 - c}{a} + \left(\frac{b}{2a}\right)^2}$$

It can be generalized to the t_n value corresponding to the crossing of the *n*-th level s_n.

$$t_n = -\frac{b}{2a} + \sqrt{\frac{s_n - c}{a} + \left(\frac{b}{2a}\right)^2}$$

The level-crossing amplitudes can have arbitrary values. For an equidistant case we have

$$s_n = s_1 + (n-1)\delta$$

Given a, b, c, s_1, and δ, one can find all the level-crossing time values.

For signal segments represented as quadratic, the time value t_n corresponding to a given level s_n is given by

$$t_n = -\frac{b}{2a} + \sqrt{\frac{s_n - c}{a} + \left(\frac{b}{2a}\right)^2},$$

For equidistant levels, $\qquad s_n = s_1 + (n-1)\delta$

Figure 1 shows a signal built from three quadratic functions. The level crossings and the corresponding time values have also been given.

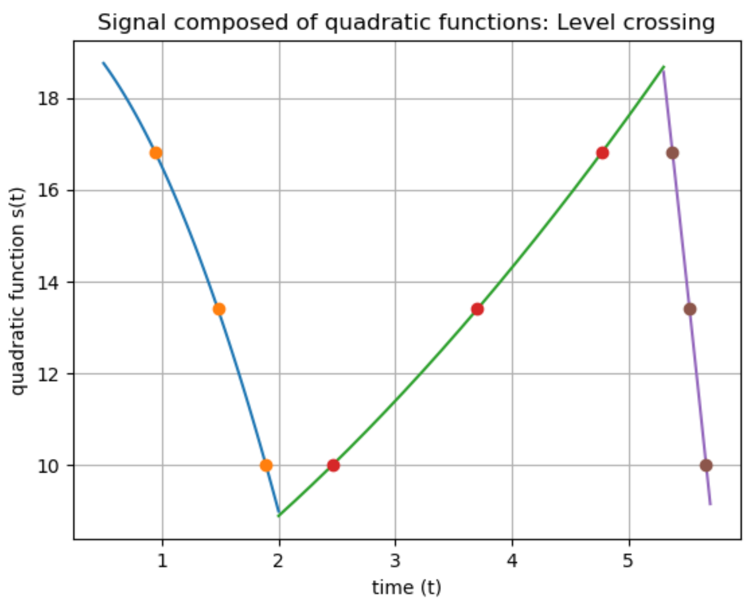

Figure 1. Three quadratic curves stitched as parts of a signal. The time values for a chosen level-crossing are the dots on the curves.

In Figure 1, the three quadratic curves have the following parameters:

- First quadratic curve: c = 20, b = –1.5, a = –2
- Second quadratic curve: c = 5.1, b = 1.5, a = 0.2
- Third quadratic curve: c = 82.69, b = –1.5, a = –2

The plot gives the time values corresponding to the three equidistant levels. This illustrates that a general signal parametrized as a sequence of many quadratics can yield exact time values using the given analytical approach.

4.2 Signal Represented as Piecewise Cubic Functions

Let s_i denote the levels at time t and be represented as a cubic. The first few levels are

$$s_1 = at_1^3 + bt_1^2 + ct_1 + d$$

$$s_2 = at_2^3 + bt_2^2 + ct_2 + d$$

$$\ldots..$$

$$s_n = at_n^3 + bt_n^2 + ct_n + d$$

The first equation can be rewritten as

$$t_1^3 + \frac{b}{a}t_1^2 + \frac{c}{a}t_1 - \left(\frac{s_1 - d}{a}\right) = 0$$

It can be converted to a *depressed cubic* as

$$\left(t_1 + \frac{b}{3a}\right)^3 + p\left(t_1 + \frac{b}{3a}\right) - \left(\frac{s_1}{a} - q\right) = 0$$

where

$$p = \frac{c}{a} - 3\left(\frac{b}{3a}\right)^2, q = \frac{d}{a} - \left(\frac{c}{a}\right)\left(\frac{b}{3a}\right) + 2\left(\frac{b}{3a}\right)^3$$

The solution for t_1 corresponding to the first level-crossing becomes

$$t_1 = -\frac{b}{3a} + \left[\frac{1}{2}\left(\frac{s_1}{a} - q\right) + \Delta_1\right]^{\frac{1}{3}} + \left[\frac{1}{2}\left(\frac{s_1}{a} - q\right) - \Delta_1\right]^{\frac{1}{3}}, \Delta_1 = \sqrt{\frac{1}{4}\left(\frac{s_1}{a} - q\right)^2 + \left(\frac{p}{3}\right)^3}$$

It can be generalized to the t_n value corresponding to the *n*-th level-crossing.

$$t_n = -\frac{b}{3a} + \left[\frac{1}{2}\left(\frac{s_n}{a} - q\right) + \Delta_n\right]^{\frac{1}{3}} + \left[\frac{1}{2}\left(\frac{s_n}{a} - q\right) - \Delta_n\right]^{\frac{1}{3}}$$

$$\Delta_n = \sqrt{\frac{1}{4}\left(\frac{s_n}{a} - q\right)^2 + \left(\frac{p}{3}\right)^3}$$

The level-crossing amplitudes can have arbitrary values. For an equidistant case we have

$$s_n = s_1 + (n-1)\delta$$

Given a, b, c, d, s_1, and δ, one can find all the level-crossing time values.

For signals represented as cubic, the time value t_n corresponding to a given level s_n is given by

$$t_n = -\frac{b}{3a} + \left[\left(\frac{s_n}{2a} - \frac{q}{2}\right) + \Delta_n\right]^{\frac{1}{3}} + \left[\left(\frac{s_n}{2a} - \frac{q}{2}\right) - \Delta_n\right]^{\frac{1}{3}}$$

where

$$\Delta_n = \sqrt{\left(\frac{s_n}{2a} - \frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3},$$

For equidistant levels, $s_n = s_1 + (n-1)\delta$

$$p = \left(\frac{c}{a}\right) - 3\left(\frac{b}{3a}\right)^2, q = \left(\frac{d}{a}\right) - \left(\frac{c}{a}\right)\left(\frac{b}{3a}\right) + 2\left(\frac{b}{3a}\right)^3$$

Figure 2 shows a signal built from three cubic functions. The level-crossings and the corresponding time values have also been given.

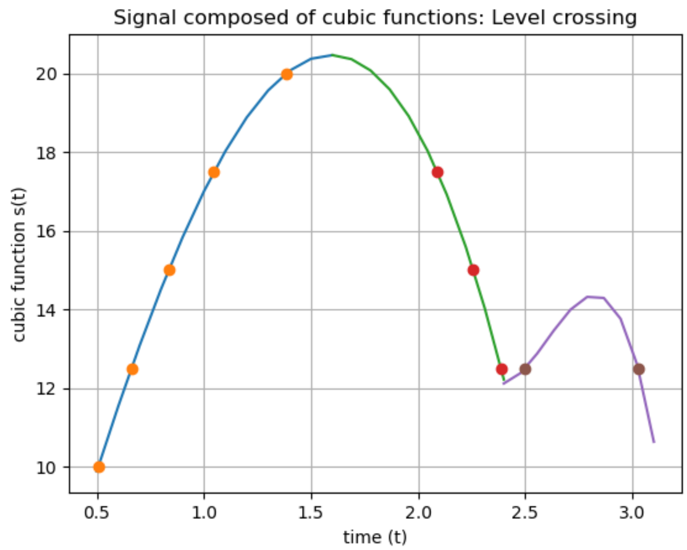

Figure 2. Three cubic curves stitched as parts of a signal. The time values for a chosen level-crossing are the dots on the curves.

In Figure 2, the three cubic curves have the following parameters:

- First cubic curve: (a, b, c, d) = (–2, –1.5, 20, 0.5)
- Second cubic curve: (a, b, c, d) = (–2, –1.5, 20, 0.5)
- Third cubic curve: (a, b, c, d) = (–52, 406, –1049, 910)

The plot gives the time values corresponding to the three equidistant levels. This illustrates that a general signal parametrized as a sequence of many quadratics can yield exact time values using the given analytical approach.

4.3 Signal Represented as Raised Cosine Function

The raised cosine function (RCF) is the basic tool for digital-to-analog conversion of a communication signal. Here a slightly modified RCF will be used to illustrate the level-crossing calculation.

The continuous signal is expressed as a sum over the combination of discrete values and RCF.

$$x_d(t) = \sum_{n=1}^{\infty} x_d[n] \frac{\cos(\pi\beta t)}{1-4\beta^2 t^2} \cdot \frac{\sin(\pi t)}{(\pi t)}, t = \tau_0 + n\Delta t$$

Here β is the roll-off factor.

We will use a few RCF terms to illustrate the method presented here. Let s_i denote the levels at time t and then the first few levels are

$$s_i = \frac{cos(\pi\beta t_i/T_s)}{1-4\beta^2(t_i/T_s)^2} \cdot \frac{sin(\pi t_i/T_s)}{(\pi t_i/T_s)} + c, i = 1,2,3,\ldots$$

The constant c has been added to avoid the negative values for the sake of illustration. The subscript i in τ_i refers to different points on a single curve.

The solution for time cannot be obtained in a closed analytical form. Because of that, we will use a cubic function, which reproduces four points denoting level-crossing on the curve.

$$s_i = at_i^3 + bt_i^2 + ct_i + d, \qquad i = 1,2,3,4$$

Figure 3 shows a modified RCF signal and the level-crossings. The corresponding time values have also been given.

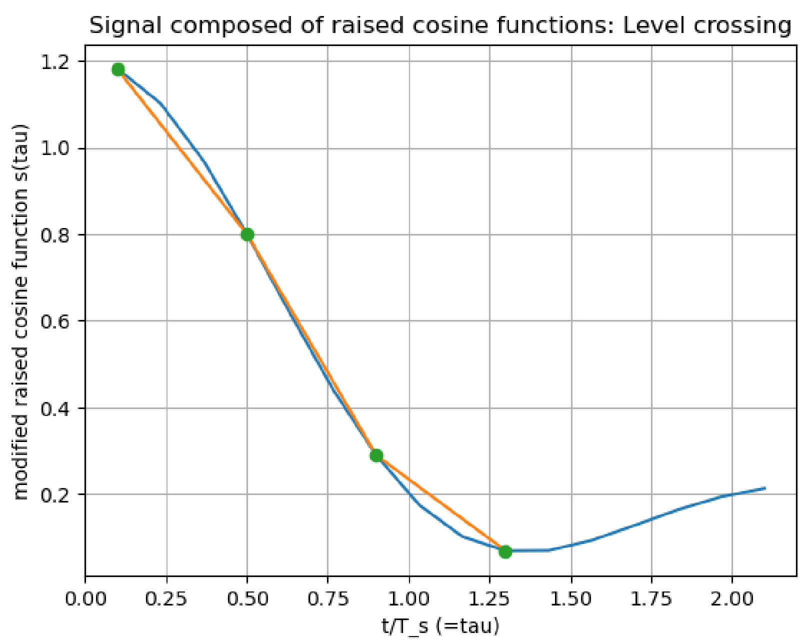

Figure 3. A modified raised cosine function representing an arbitrary impulse function (in blue). The orange curve is a cubic that intersects the RCF and mostly approximates it.

In Figure 3, the cubic approximating RCF has the following parameters:

$$(a, b, c, d) = (1.094, –2.047, –0.0609, 1.2055)$$

Level-crossings occur at

$$(\tau, s) = (0.1, 1.8), (0.5, 0.8), (0.9, 0.44), (1.3, 0.1), \beta = 0.5$$

The plot shows that an RCF can be well approximated as a series of stitched cubic functions with as much fidelity as needed. The method should be particularly more useful when the signal is band-limited. The interpolation using cubic spline (e.g., scipy.interpolate.CubicSpline program in Python) is a more refined example of the approach presented here, but it is computationally more intensive compared to the current one.

5. Conclusion and Future Directions

In this work, some analytical approaches have been presented for calculating the nonuniform time values corresponding to level-crossings of a band-limited signal. The method needs fewer computational resources than standard algorithms. In the future we will compare both approaches in more detail and ascertain their relative usefulness when applied to the level-crossing determination of band-limited continuous time signals.

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List of Symbols, Abbreviations, and Acronyms

CT-DSP	continuous-time digital signal processing
DSP	digital signal processing
DT-DSP	discrete-time digital signal processing
ECG	electrocardiogram
LCD	level-crossing delta
LCS	level-crossing sampled
ML	machine learning
RCF	raised cosine function
SINAD	signal-to-noise and distortion
SNN	spiking neural network
SQNR	signal-to-quantization noise ratio

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