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On Using Cubic Interpolation for Level-Crossing Sampling Continuous-Time Signals

by Vinod Mishra, W. Michael Crowe, and Patrick Jungwirth

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Vinod Mishra and Patrick Jungwirth
DEVCOM Army Research Laboratory

W. Michael Crowe
DEVCOM Aviation and Missile Center

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14. ABSTRACT
Signals (e.g., communication, radar, and electrocardiogram) in general are analyzed using discrete-time methods. Recently, advantages of continuous-time methods for ML have been observed. This approach is nonuniform in time and uses sampling based on level crossing. We present an analytical model using cubic functions for the level-crossing-based digital sampling process.

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Vinod Mishra	(520) 691-5761

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1. Introduction

Signals in the tactical arena comprise many kinds with characteristics varying across a wide range, e.g., communication, sensing, and radar signals. Signal-processing approaches applicable to these signals use either discrete- or continuous-time sampling. The last approach based on continuous time (CT) is a relatively new one and has been shown to offer distinct advantages in digital signal processing (DSP) and analysis of physiological signals, e.g., electrocardiogram (ECG).[1–7] In 2025, an analytical approach was presented to calculate the level crossing of these signals.[8] In the current work, we extend that approach to concatenated cubic curves representing the level-crossing sampled (LCS) signals.

2. Continuous-Time DSP (CT-DSP)

Discrete-time DSP (DT-DSP) is the standard approach to signal processing. It uses ideas such as Nyquist rate and cardinal sine (sinc) functions. The other approach is CT-DSP,[9–13] which is a relatively new subfield of signal processing. It has time domain properties like analog signal processing (asynchronous, no clock) with the benefits of DSP without the limitations of conventional sampling. It has many advantages over the conventional DT-DSP, including the following.

- The input CT signal is quantized when it exactly equals a voltage level, resulting in no inherent quantization error. Power savings due to this can be as high as 10× in some cases.
- The sampling frequency of the input CT signal is proportional to its slope; e.g., a 2.5-s ECG dataset requires only 225 samples, compared with 1250 samples for conventional DSP. Fewer samples result in less data processing and lower energy.
- It does not use uniform time sampling, so it has no frequency aliasing.
- It uses adaptive sampling, so an input signal is only captured when it crosses a threshold level. For signals whose important amplitudes are known as a priori, this can lead to significant power savings.
- The input signal is captured at the exact time and amplitude at which it crosses a threshold level. This accuracy allows the potential for high signal-to-noise and distortion reconstruction of LCS signals (LCS is a sequence of Dirac-delta functions exactly representing the time and amplitude of all detected level crossings).

The time accuracy of the sampling benefits control systems and allows accurate detection of the transient signals. This is accomplished by giving information at the time of a transient, instead of waiting for the next clock tick. Both CT-DSP and analog signal processing are asynchronous (no clock), but the former has a nonuniform time step.

3. Level Crossing and a CT Signal

CT-DSP has discrete amplitude and CT and emphasizes accuracy in timing over that in amplitude. It is characterized by times t_k, when the input signal exactly equals a discrete amplitude and there is no quantization error compared with DSP. It is nonlinear so the signal reconstruction must be done before any processing. We will consider the case of ideal digital sampling without quantization.

The CT sample point is a vector given as either of the two forms:

- The *k*-th time stamp and amplitude level, (t_k, a_k) or
- a time difference and an amplitude level $(t_k - t_{k-1}, a_k)$.

The time steps are fixed for the DT-DSP but are nonuniform for the CT-DSP. Input signal vectors in such a system can be characterized in either of the following two ways:

- (t_k, a_k) where t_k= *k*-th level-crossing time stamp, or
- $(\Delta t_k, a_k)$ where Δt_k= $t_{k+1} - t_k$.= time difference between the *k* + *1*-th and *k*-th level-crossing time stamps, and
- $a_k = n_k \Delta V$= the *k*-th amplitude level (n_k = an integer, and ΔV = the distance between voltage levels).

For CT, $x_{CT}(t_k) = a_k$ = the amplitude level for level crossing at time t_k.

For DT, $x_{DT}(k) = x(t_k) = x(kT)$, *k* is an integer, and *T* is the sample spacing.

The following relation connects a DT signal $x_{DT}[n]$ to a corresponding CT signal $x_{CT}(t)$:

$$x_{CT}(t) = \sum_k x_{DT}(k) p(\tau_0 + k\Delta t), t = \tau_0 + k\Delta t \tag{1}$$

Here, τ_0 is a constant, k is an integer, and Δt is the time step. The *p*-function is an analytical function and depends on the nature and properties of the signal. For example, raised cosine function is used for band-limited signals. Regular level-crossing values in a CT signal correspond to nonuniform time values in $x_d(t)$.

4. An Analytical Approach to the Label-Crossing-Based Sampling

The level crossing applied to a complex signal produces segments that may have arbitrary shapes. To understand their properties, one must approximate them by an analytic function. The cubic is a sufficiently complex function to lead to such an understanding and, at the same time, is easy enough to implement. In the following sections, they are used to represent signal segments obtained after level crossing has been applied to them.

4.1 Signal Represented as Two Cubic Functions with Equal Slopes

The slopes of the segments were not considered separately in the 2025 approach.[8] Now we impose the conditions that slopes of the segments are equal at the intersection.

4.1.1 First and Second Segments

The first cubic and its slope are given as

$$s = a_1 t^3 + b_1 t^2 + c_1 t + d_1 \tag{2}$$

Let (t_1, s_1), (t_2, s_2), (t_3, s_3), and (t_4, s_4) be the four points on the first curve. The four cubic equations are solved (as given in the Appendix) to obtain four unknowns (a_1, b_1, c_1, d_1) as shown earlier.

The slope of the first cubic function (hereafter shortened to "cubic") is given as

$$\frac{ds}{dt} = s' = 3a_1 t^2 + 2b_1 t + c_1 = k_1 \,. \tag{3}$$

Similarly, the second cubic and its slope are

$$s = a_2 t^3 + b_2 t^2 + c_2 t + d_2$$

$$\frac{ds}{dt} = s' = 3a_2 t^2 + 2b_2 t + c_2 \,. \tag{4}$$

The point (s_4, t_4) is common to both curves. The continuity condition is captured by requiring that at the meeting point t_4, the values of the two cubics are equal:

$$s_4 = a_2 t_4^3 + b_2 t_4^2 + c_2 t_4 + d_2 \,. \tag{5}$$

The slopes of the two cubics are equal.

$$k_1 = 3a_2t_4^2 + 2b_2t_4 + c_2 \; . \qquad (6)$$

Thus, equating the value and the slope at the meeting point reduces the number of unknowns to two for the second cubic. The unknown coefficients (a_2, b_2) of the second cubic are found by solving the following matrix equation applied to the subsequent two points (s_4, t_4) and (s_4, t_4).

$$\begin{bmatrix} t_5 + 2t_4 & 1 \\ t_6 + 2t_4 & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} \{(s_5 - s_4) - k_1(t_5 - t_4)\}/(t_5 - t_4)^2 \\ \{(s_6 - s_4) - k_1(t_6 - t_4)\}/(t_6 - t_4)^2 \end{bmatrix} \qquad (7)$$

The other two coefficients are found from

$$d_2 = s_4 - (a_2t_4^3 + b_2t_4^2 + c_2t_4) \qquad (8)$$

$$c_2 = k_1 - 3a_2t_4^2 - 2b_2t_4 \; . \qquad (9)$$

The above four coefficients (a_2, b_2, c_2, d_2) define the second cubic curve.

4.1.2 Example

For the first cubic curve, we start with four points with coordinates given as

(t_1, s_1) = (2.4, 12), (t_2, s_2) = (2.6, 13), (t_3, s_3) = (2.8, 14), and (t_4, s_4) = (3, 12.5) .

This leads to the coefficients of the first cubic curve

$$(a_1, b_1, c_1, d_1) = (–52.08, 406.25, –1049.17, 910) \qquad (10)$$

with the slope as

$$k_1 = 3a_1t_4^2 + 2b_1t_4 + c_1 = –17.92 \; . \qquad (11)$$

For the second cubic curve, we start with three points with coordinates given as

$$(t_4, s_4) = (3, 12.5), (t_5, s_5) = (3.2, 12), \text{ and } (t_6, s_6) = (3.4, 10) \; . \qquad (12)$$

The value and the slope at the meeting point (t_4, s_4) are equated leading to the (a_2, b_2) coefficients of the second cubic curve. The other two coefficients (c_2, d_2) are derived using them. Finally, the coefficients are

$$(a_2, b_2, c_2, d_2) = (–239.58, 2281.25, –7236.67, 7660) \; . \qquad (13)$$

Figure 1 shows the two blue and orange color curves with a common point at (t_4, s_4) = (3,12.5). As expected, six points are needed for this scheme to derive all the coefficients of the two cubic curves. Given any level-crossing values, the corresponding times can be obtained by using the cubic root expressions.

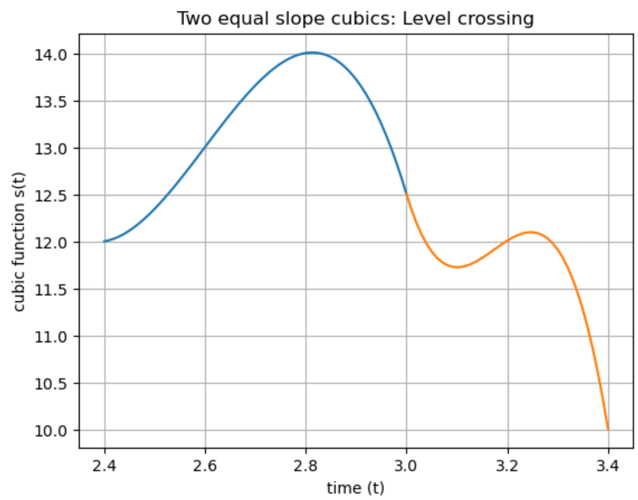

Figure 1. **Two cubic curves with continuous value and slope at the meeting point.**

4.2 Continuation of the Scheme to the Third Cubic

Now we extend this approach to the third cubic after the first two.

4.2.1 Third Segment

Let us recall that the second cubic and its slope:

$$s = a_2 t^3 + b_2 t^2 + c_2 t + d_2 \tag{14}$$

$$\frac{ds}{dt} = s' = 3a_2 t^2 + 2b_2 t + c_2 \ . \tag{15}$$

The previous process is now extended to the third cubic with the common point at (t_6, s_6). The continuity condition is captured by requiring that at the common point, the values of the two cubics are equal:

$$s_6 = a_3 t_6^3 + b_3 t_6^2 + c_3 t_6 + d_3 \ . \tag{16}$$

The slopes of the two cubics are equal:

$$k_2 = 3a_3 t_6^2 + 2b_3 t_6 + c_3 \ . \tag{17}$$

The unknown coefficients (a_3, b_3) of the third cubic are found by solving the following matrix equation applied to the subsequent two points (t_7, s_7) and (t_8, s_8):

$$\begin{bmatrix} t_7 + 2t_6 & 1 \\ t_8 + 2t_6 & 1 \end{bmatrix} \begin{bmatrix} a_3 \\ b_3 \end{bmatrix} = \begin{bmatrix} \{(s_7 - s_6) - k_2(t_7 - t_6)\}/(t_7 - t_6)^2 \\ \{(s_8 - s_6) - k_2(t_8 - t_6)\}/(t_8 - t_6)^2 \end{bmatrix} \tag{18}$$

The other two coefficients are found from

$$d_3 = s_6 - (a_3 t_6^3 + b_3 t_6^2 + c_3 t_6) \tag{19}$$

$$c_3 = k_2 - 3a_3 t_6^2 - 2b_3 t_6 \,. \tag{20}$$

The above four coefficients (a_3, b_3, c_3, d_3) define the third cubic curve.

4.2.2 Example

For the third cubic curve, we start with three points with coordinates given as

$$(t_6, s_6) = (3.4,\ 10),\ (t_7, s_7) = (3.6,\ 9), \text{ and } (t_8, s_8) = (3.8,\ 11)\,. \tag{21}$$

The value and the slope at the meeting point (t_6, s_6) are equated leading to the (a_3, b_3) coefficients of the third cubic curve. The other two coefficients $(c_3, d_3$ are derived using them. So finally, the coefficients are

$$(a_3, b_3, c_3, d_3) = (–255.208,\ 2793.75,\ –10{,}179.792,\ 12{,}356.25)\,. \tag{22}$$

Figure 2 shows the two orange and green curves with a common point at (t_6, s_6) = (3.4, 10).

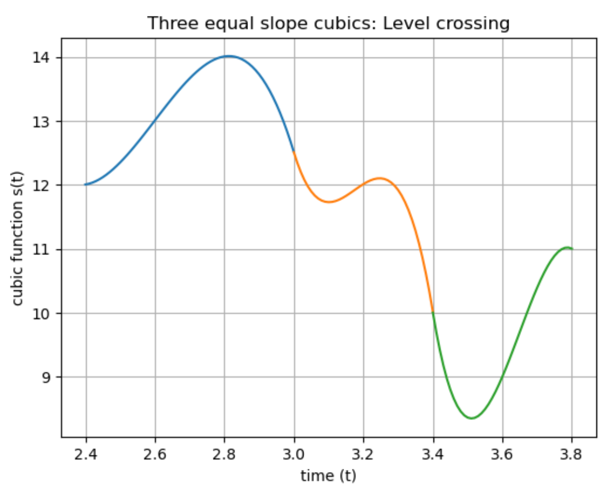

Figure 2. Three cubic curves with continuous value and slope at the meeting points.

This scheme can be repeated as many times as needed to find a CT envelope of a DT signal. The signal can be periodic or nonperiodic in time because of the analytic nature of the cubic curve. The final expression can be used to calculate the time value corresponding to any desired level crossing by finding the roots of a cubic equation.

4.2.3 Generalization of the Procedure

The steps of the process applicable to this approach are as follows:

1. Start with the first four level-crossing points given as (t_i, s_i), $i = 1, \cdots, 4$. The time steps can be either uniform or arbitrary.
2. Obtain the coefficients (a_1, b_1, c_1, d_1) of the cubic curve passing through these points by solving the four cubic equations passing through the four points given earlier.
3. Calculate the point and slope values of the first cubic at the common point (t_4, s_4) using its coefficients.
4. Identify the three points through which the second cubic will pass. The first such point (t_4, s_4) is the last point used for the first cubic. The next two points are (t_5, s_5) and (t_6, s_6).
5. Use the results of step 3 to reduce the number of unknowns for the second cubic coefficients to (a_2, b_2). The other coefficients (c_2, d_2) can be obtained from using the results of step 3. Solve the equations to get (a_2, b_2) and afterwards (c_2, d_2).
6. Repeat steps 3–5 to get the third and other cubic curves as needed.

4.3 Signal Represented as Piecewise Aliased Sine Cardinal (asinc) Functions and Improving the Fit

The asinc functions are used in the reconstruction of the CT signals from DT signals. They are the basis of the digital-to-analog conversion of the signals and are also known as LCS signals.

Given the fundamental frequency ω_0, the waveform period is

$$T_p = (2\,\pi/\omega_0)q \tag{23}$$

and sample spacing is

$$T_s = \left(\frac{2\pi}{\omega_0}\right)\left(\frac{q}{N}\right) = \frac{T_p}{N} \tag{24}$$

such that $T_s < (\pi/\omega_{max})$ and q are integers (usually 1).

The signal is represented as

$$s(t) = \sum_{n=0}^{N-1} s_n h(t - nT_s)\ . \tag{25}$$

Let $\alpha = \frac{\pi}{T_s}$, then for even N,

$$h(t) = \left(\frac{1}{N}\right)\frac{cos\,(\pi t/T_p)}{sin\,(\pi t/T_p)} sin\,(\pi t/T_s) = \left(\frac{1}{N}\right)\frac{cos\,(\alpha t/N)}{sin\,(\alpha t/N)} sin\,(\alpha t) \qquad (26)$$

So the signal becomes

$$s(t) = \sum_{n=0}^{N-1} s_n \left(\frac{1}{N}\right) \cos((\alpha t - n\pi)/N) \qquad (27)$$

$$\frac{1}{sin\,((\alpha t - n\pi)/N)} sin\,(\alpha t - n\pi)\,. \qquad (28)$$

Figure 3 presents the full asinc signal approximated by a cubic. The two curves follow one another for the initial one-third of the total time, diverge afterwards, and meet again at the end. This shows that the cubic is a good approximation to the theoretical curve.

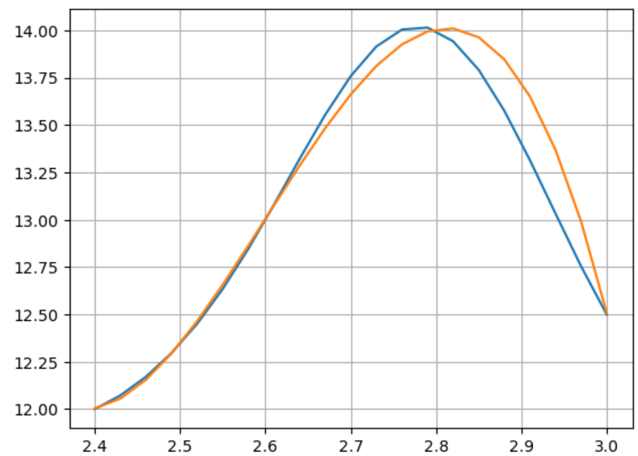

Figure 3. The signal expressed in terms of asinc functions (N = 4, T_s = 0.2, orange curve) compared with the first cubic of Figure 2 (blue curve).

One can find a better match by refitting a new cubic to the asinc curve in the region where they differ. This becomes possible due to the analytic nature of this approach. Table 1 presents the result of this approach.

Table 1. Parameters of the cubic curves.

Segment No.	Range	(*t*,*s*) values	Equal slope point	Cubic coefficients
First	(2.4, 12) - (2.6, 13)	(t_1, s_1) = (2.4, 12), (t_2, s_2) = (2.467, 12.527), (t_3, s_3) = (2.534, 12.733), (t_4, s_4) = (2.6, 13).	First cubic slope: no constraint	(a_1, b_1, c_1, d_1) = (215.267, 1628.949, 4111.328, 3448.296)
Second	(2.6, 13) - (2.8, 14)	(t_4, s_4) = (2.6, 13), (t_5, s_5) = (2.7, 13.299), (t_6, s_6) = (2.8, 14).	(t_4, s_4) = (2.6, 13)	(a_2, b_2, c_2, d_2) = (122.902, 976.176, -2577.256, 2275.039)
Third	(2.8, 14) - (3, 12.5)	(t_6, s_6) = (2.8, 14), (t_7, s_7) = (2.9, 13.299), (t_8, s_8) = (3, 12.5).	(t_6, s_6) = (2.8, 14)	(a_3, b_3, c_3, d_3) = (258.728, -255.934, 6546.62, -6309.614)
Fourth	(3, 12.5) - (3.2, 12)	(t_8, s_8) = (3, 12.5), (t_9, s_9) = (3.1, 12.356), (t_{10}, s_{10}) = (3.2, 12).	(t_8, s_8) = (3, 12.5)	(a_4, b_4, c_4, d_4) = (147.4, 1360.22, 4184.848, 4304.864)
Fifth	(3.2, 12) - (3.4, 10)	(t_{10}, s_{10}) = (3.2, 12), (t_{11}, s_{11}) = (3.3, 10.807), (t_{12}, s_{12}) = (3.4, 10).	(t_{10}, s_{10}) = (3.2, 12)	(a_5, b_5, c_5, d_5) = (314.6, 3095.24, 10137.456, 11041.414)

The asinc curve with the *t*-range from $t = 2.4$ to $t = 3.4$ is divided into five segments. The cubics fitting them are found such that their slopes are equal at their meeting points. The algorithm presented earlier is used repeatedly for this purpose.

The cubics provide a good fit overall, as shown in Figure 4. The algorithm can be changed if better fits are needed for smaller segments.

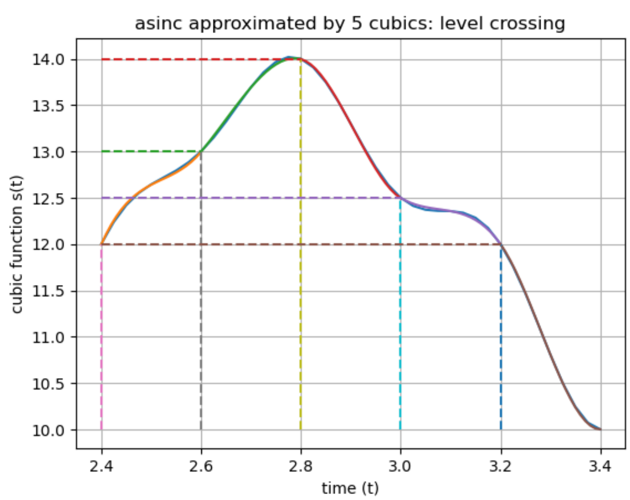

Figure 4. The larger range signal expressed in terms of asinc functions and its approximation by five cubic curves.

5. Conclusion and Future Directions

This work improves upon the 2025 approach[8] to give an analytical method to obtain a CT envelope to discrete signals. The examples focus on the time-periodic discrete signals, but the method can be easily extended to the nonperiodic case and needs less computational resources than standard such algorithms.

In the future, we will include noise and compare this approach with others derived from those that use better theoretical justification but are difficult to use analytically. We will also explore the computational cost savings from this approach in edge computing compared with those used currently.

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Appendix. Roots of Cubic Equation

Let s_i denote the levels at time t_i and be represented as a cubic. The first few levels are

$$s_1 = at_1^3 + bt_1^2 + ct_1 + d \quad \text{(A-1)}$$

$$s_2 = at_2^3 + bt_2^2 + ct_2 + d \quad \text{(A-2)}$$

$$\cdots$$

$$s_n = at_n^3 + bt_n^2 + ct_n + d\ . \quad \text{(A-3)}$$

The first equation can be rewritten as

$$t_1^3 + \frac{b}{a}t_1^2 + \frac{c}{a}t_1 - \left(\frac{s_1 - d}{a}\right) = 0\ . \quad \text{(A-4)}$$

It can be converted to a *depressed cubic* as:

$$\left(t_1 + \frac{b}{3a}\right)^3 + p\left(t_1 + \frac{b}{3a}\right) - \left(\frac{s_1}{a} - q\right) = 0 \quad \text{(A-5)}$$

where

$$p = \frac{c}{a} - 3\left(\frac{b}{3a}\right)^2,\ q = \frac{d}{a} - \left(\frac{c}{a}\right)\left(\frac{b}{3a}\right) + 2\left(\frac{b}{3a}\right)^3\ . \quad \text{(A-6)}$$

The solution for t_1 corresponding to the first level crossing becomes

$$t_1 = \frac{-b}{3a} + \left[\frac{1}{2}\left(\frac{s_1}{a} - q\right) + \Delta_1\right]^{\frac{1}{3}} + \left[\frac{1}{2}\left(\frac{s_1}{a} - q\right) - \Delta_1\right]^{\frac{1}{3}},\ \Delta_1 = \sqrt{\frac{1}{4}\left(\frac{s_1}{a} - q\right)^2 + \left(\frac{p}{3}\right)^3} \quad \text{(A-7)}$$

It can be generalized to the t_n-value corresponding to the *n*-th level crossing.

$$t_n = \frac{-b}{3a} + \left[\frac{1}{2}\left(\frac{s_n}{a} - q\right) + \Delta_n\right]^{\frac{1}{3}} + \left[\frac{1}{2}\left(\frac{s_n}{a} - q\right) - \Delta_n\right]^{\frac{1}{3}} \quad \text{(A-8)}$$

$$\Delta_n = \sqrt{\frac{1}{4}\left(\frac{s_n}{a} - q\right)^2 + \left(\frac{p}{3}\right)^3} \quad \text{(A-9)}$$

The level-crossing amplitudes can have arbitrary values. For equidistant case we have

$$s_n = s_1 + (n-1)\delta\ . \quad \text{(A-10)}$$

Given a, b, c, *d*, s_1, and δ, one can find all the level-crossing time values.

For signals represented as cubic, the time-value t_n corresponding to a given level s_n is given by

$$t_n = \frac{-b}{3a} + \left[\left(\frac{s_n}{2a} - \frac{q}{2}\right) + \Delta_n\right]^{\frac{1}{3}} + \left[\left(\frac{s_n}{2a} - \frac{q}{2}\right) - \Delta_n\right]^{\frac{1}{3}} \qquad \text{(A-11)}$$

where

$$\Delta_n = \sqrt{\left(\frac{s_n}{2a} - \frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}\ . \qquad \text{(A-12)}$$

For equidistant levels,

$$s_n = s_1 + (n-1)\delta \qquad \text{(A-13)}$$

$$p = \left(\frac{c}{a}\right) - 3\left(\frac{b}{3a}\right)^2, q = \left(\frac{d}{a}\right) - \left(\frac{c}{a}\right)\left(\frac{b}{3a}\right) + 2\left(\frac{b}{3a}\right)^3\ . \qquad \text{(A-14)}$$

List of Symbols, Abbreviations, and Acronyms

asinc	Aliased Sine Cardinal
CT	Continuous Time
CT-DSP	Continuous-Time Digital Signal Processing
DSP	Digital Signal Processing
DT	Discrete Time
DT-DSP	Discrete-Time Digital Signal Processing
ECG	Electrocardiogram
LCS	Level-Crossing Sampled
ML	Machine Learning
sinc	Cardinal Sine

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